Conformal invariance and universality in the 2D Ising model
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It is conjectured that many 2D lattice models of physical phenomena (percolation, Ising model of a ferromagnet, self avoiding polymers, ...) become invariant under rotations and even conformal maps in the scaling limit (i.e. when “viewed from far away”). A well-known example is the Random Walk (invariant only under rotations preserving the lattice) which in the scaling limit converges to the conformally invariant Brownian Motion.

Assuming the conformal invariance conjecture, physicists were able to make a number of striking but unrigorous predictions: e.g. dimension of a critical percolation cluster is almost surely $91/48$; the number of simple length $N$ trajectories of a Random Walk is about $N^{11/32} \cdot \mu^N$, with $\mu$ depending on a lattice, and so on.

We will discuss the recent progress in mathematical understanding of this area, in particular for the Ising model. Much of the progress is based on combining ideas from probability, complex analysis, combinatorics.